Bioeng 3070/5070

Stats for Bioengineer
Lecture 15
Maximum Likelihood Estimation (MLE)

• The idea is very simple: Given probabilistic model of the data as a function of the parameter to be estimated use the value that maximizes the joint probability density of the samples!!

• The parameters are assumed to be unknown deterministic constants.
Example: Gaussian

• Let $x_1, x_2, \cdots, x_n$ be a random sample of size $n$ from a Gaussian distribution with mean $\mu$ and variance $\sigma^2$, find the maximum likelihood estimates of mean and the variance.

• The joint probability density function is given by:

$$f(x_i=1\ldots,n \mid \mu, \sigma) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$
Example Gaussian:

- We maximize the above function:
- (Blackboard)

- The MLE estimator of the mean and the variance is given by:

\[
\hat{\mu} = \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \hat{\sigma}^2 = \frac{1}{n} \sum_{i} (x_i - \bar{x})^2
\]
Bayesian Estimation

- In Bayesian estimation the parameters assumed to be random variables themselves with prior-distributions, reflecting the strengths of one’s belief about possible values they can assume.

- The estimation problem is formulated as that of finding the posterior distribution of the parameters given the data.
Bayesian Estimation

• Use Bayes’ Rule to find the posterior given the data model and the prior distribution.

\[ P(\theta|X) = \frac{p(\theta)p(x|\theta)}{p(x)} \]

\( p(\theta) \): Prior Distribution

\( P(\theta|x) \): The posterior