Discrete Probability Distributions

• Beginning Reading Chapter 4
• Overview and Introduction
  – Last chapter, probability of specific events
  – This chapter, probability models based on the probabilities of certain events from past data.
Random Variable

• Let S be a sample space with a probability measure P.
• A real random variable $x$ is a process of assigning a real number $x(s)$ to every outcome $s \in S$.
• Two types: Discrete and Continuous
Example

• Rolling a dice where each side is one color
  Sample space \( (S) = \) each color \( (s) \)
  \[
  x(s) = \begin{cases} 
  1 & s = \text{red} \\
  2 & s = \text{blue} \\
  \ldots & \ldots \\
  6 & s = \text{green} 
  \end{cases}
  \]

• Discrete set of values with specified probabilities
Example

• Number of heads when tossing 4 coins.
  \[ S = \text{the set of all possible outcomes (s)} \]
  \[
  \begin{align*}
  0 & \quad s = \{TTTT\} \\
  1 & \quad s = \{HHTT, THTT, TTHT, TTTT\} \\
  2 & \quad s = \{HHTT, HTHT, HTTH, THTH, TTHH, TTHT\} \\
  3 & \quad s = \{THHH, HTHH, HHTH, HHHT\} \\
  4 & \quad s = \{HHHH\}
  \end{align*}
  \]

• Discrete set of values with specified probabilities
Probability-Mass Function

• If X is a discrete random variable, the function given by \( f(x) = p(x=x) \) for each \( x \) with in the range of \( x \) is called the probability distribution of \( x \)
• Probability of an event is between 0 and 1
• Sum of all the probabilities is exactly 1
Probability-Mass Function

• 6 Color Die Example

\[ F(x) = \begin{cases} 
1/6 & x = \{\text{red, blue, ..., green}\} \\
0 & \text{otherwise}. 
\end{cases} \]
• 4 Coin Example

\[
F(x) = \begin{cases} 
1/16 & \text{if 0 heads} \\
4/16 & \text{if 1 head} \\
6/16 & \text{if 2 heads} \\
4/16 & \text{if 3 heads} \\
1/16 & \text{if 4 heads} \\
0 & \text{otherwise}
\end{cases}
\]
Experiment With 4 Coins

• As more tests are preformed, what is the relationship between the actual frequency of the test and the probability distribution?
• What does the probability-mass distribution look like with more coins in each toss? An infinite number of coins?
Probability and Frequency Distributions

• Frequency distribution is the actual sample proportion
• Probability distribution can be thought of as a frequency distribution with an infinitely large sample
• Goodness-of-fit test is used to compare the observed sample-frequency to the probability distribution.
Expected Value

- For a discrete random variable

\[ E(x) \equiv \mu = \sum_{i=1}^{R} x_i P(X=x_i) \]

- What is the expected value of a dice?
Variance

• Analogous to sample variance \((s^2)\)
• For a discrete random variable \((X)\) with mean \(\mu\)

\[
\text{Var}(X) = \mathbb{E}[(X - \mu)^2].
\]
95% Rule

• Approximately 95% of the probability mass falls within 2 sigma
Cumulative-Distribution Function (CFD)

• A CFD of a discrete random variable $X$ is denoted by $F(X)$ and for a specific value $x$ of $X$, is defined by $P(X \leq x)$ and denoted by $F(x)$

• Remember big $X$ is the set of all $x$ values

• Example
Permutations and Combinations

• Permutations – number of unique sequences of elements selected from a finite set
  
  $N = \text{number of items}, \; r = \text{number to be selected}$

  $$P(n, r) = \frac{n!}{(n - r)!}.$$  

  If $N = R$ then how many permutations do you get?
Permutations and Combinations

• Combinations – number of unique combinations regardless of the order selected from a finite set
  \[ C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}. \]

  If all four elements are selected at once and order doesn’t matter then how many combinations are there?

  If 3 objects are selected out of 7 and the order doesn’t matter then how many combinations are there?